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How Player Winning Rates and Average Cards to Bust are Affected by Incremented Decks in Blackjack

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Abstract

In this research paper, we investigated the impact of adding increments from 0 to 20 with a step of 0.01 to each card in a deck of Blackjack (e.g. an increment of 0.01 would add 0.01 to every card in the deck). Our focus was on how this affected the average number of cards it took for a player to bust and their win rate. Blackjack, a popular card game played in casinos for centuries, requires players to get as close as possible to 21 without going over to beat the dealer.Typically, games will have 2 to 7 players, but in this paper, the simulation analyzed a singular player versus the house. Though Blackjack includes intricacies regarding splitting hands and strategies with money, these rules will be looked over for the sake of complexity. By conducting thousands of simulations for each increment, we analyzed patterns between the increment value and the two variables of interest. Our results indicated that the player's winning percentage was highest when an increment between 9 and 11 was added, with a mid-60 percentile chance of winning. Additionally, the average number of cards required to bust approached 3.

1 Introduction

Blackjack is a highly popular card game frequently played in casinos. Its origins are subject to various theories, but the most widely accepted one is that the game first emerged in French casinos during the 1700s under the name Vingt-et-Un, which means 21 in French (Wintle, February 11, 2010). Spain also had

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a similar version of the game in which players aimed to reach 31 with at least three cards, while the Romans are believed to have played a variation involving wooden blocks with varying numerical values (Wintle, February 11, 2010).

2 Background

2.1 Rules

In typical casinos, multiple decks of cards are shuffled together, with the six-deck game being the most popular. In Blackjack, the ace card can be counted as either 1 or 11, and it's up to the player to decide which value will allow them to reach a sum closest to 21 without going over. The face cards, including jack, queen, and king, are worth 10 points each, while the numerical cards hold their face value.

2.2 Gameplay

In Blackjack, the dealer initially deals two cards face up to the player and one card face up and another face down for themselves.The player can then choose to hit, meaning they'll receive another card in an attempt to get as close to 21 without exceeding it. Once the player decides to stand, meaning they're satisfied with their current hand, the dealer flips over their face down card and hits until they reach a sum of at least 17, the standard rule used by most casinos. If the player's card sum is less than or equal to 21 and higher than the dealer's, the player wins. If the player goes over 21, they bust, resulting in an automatic loss. In the event that both the player and the dealer have the same card sum, the game results in a tie.

2.3 Optimal Strategy

The optimal strategy for playing Blackjack varies depending on the number of decks in use. However, this study focuses on a single-deck game. In this case, the player's optimal strategy involves standing if the sum of their cards ranges from 17 to 21. If the sum of their cards ranges from 13 to 16 and the dealer's face-up card is 2 to 6, or if the sum of their cards is 12 and the dealer's face-up card ranges from 4 to 6, the player should also stand. Otherwise, the player should hit, requesting another card until they fall into one of the standing ranges. The house's strategy, on the other hand, is to always hit until their sum is at least 17.

3 Literature review

There is minimal past research conducted on using different decks when playing Blackjack. However, current games can be played with one or multiple decks, changing up the strategy a player uses (Betway). The number of decks drastically changes the chances of obtaining certain hands, like Blackjack 4.83 percent for one deck to 4.78 percent for 2 decks (review fix). Studies have been conducted regarding winning chances and the house's edge (winning percentage over the player) in relation to the number of decks used. A study by the Wizard of Odds demonstrated that as the number of decks in the game increases, so do the player's winning chances, as the houses' edge decreased by 0.079 percent when the decks increased from 1 to 8 (wizard of odds). The author postulated this was because the bust rate decreased as the number of decks increased, thus increasing the chances that the player would obtain a hand better than the house's.

From searching numerous research journals, there does not seem to have been past research regarding how winning chances change when the whole deck of cards is shifted upwards. However, given the number of decks in play affects the winning chances as the bust rate changes, it can be postulated that the winning chances would also change as the decks are shifted upwards. This is because it will become easier to bust with larger card numbers. We will also measure the average number of cards it takes to bust to see if there is a correlation between the bust rate and winning rate that was true for changing the number of decks within play study.

4 Methods

As with any analysis regarding patterns in card games, it is optimal to create a simulation through code that can be automatically run many times such that it limits manual intervention. For example, instead of having to manually play through 1000 games and record the results, programming a bot to run through one game and then looping that code 1000 times would be much faster.

Before coding, some parameters and a basis was set. Since we are focusing on win rates and the average number of cards it takes to "bust", the money aspect of the game was not principal. Thus, it was acceptable to negate coding any features regarding money in Black Jack (e.g. doubling down). A strategy such as counting cards based on the running count was also not included. Moreover, splitting hands, the term given for playing two hands at once when pairs of the same card are dealt, was also looked over to limit complexity. Though splitting hands plays a vital position in the player's monetary gain over the house, it does not drastically change the winning chances shown later in the results section. Finally, it is also crucial that a set strategy was given to both the player and the house since consistent gameplay was needed if generated data were to be interpretable. Based on the number of decks in play at one time, the player can use the different optimal strategies. The notable two being for one deck and another for 6-8 decks in play. In this study, we chose to have one deck at play though the same program implemented later can be easily altered to fit the 6-8 deck strategy. The optimal strategy for the player with one deck in play is listed below:

Figure 1: The figure above shows the optimal strategy along with the key for the player with one deck in play. Note that as established earlier, in this study we only consider two of the players moves: being able to hit and stand. Thus, the Rh, which normally stands for surrender, was treated as a stand and doubling down was treated as a hit or a stand.

The house's strategy, by conventional rule, is to hit when the sum of their cards is below 17. It is crucial to consider the situation when either the player or the house has busted but has an ace in their deck. By the Blackjack rule, the ace can hold a value of 1 or 11 based on the holder's choice. Aces are initiated to hold a value of 11. However, if either player or house busted but had an ace in their deck, the ace would then be converted to a value of 1 and the player would resume asking for cards.

Given the parameters we have defined, a bot was coded, of which we will explain the significant functions.

We started by defining lists and variables:

```
numberList = [2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10,
  10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 11, 11, 11, 11]

##variables
playerCards = []
houseCards = []
winPlayer = 0
winHouse = 0
Ties = 0
bustSum = 0
bustNum = 0
x = []
y = []
```

Figure 2: The segment above displays the lists and variables created. The playerCards and houseCards lists represents the player and house's current hand, winPlayer and winHouse, and Ties count the number of times either entity has won or tied, bustSum and bustNum sum the number of cards in the hand every time the player busts and the number of times a player busted respectively. The x and y lists are defined later when plotting the distributions. Finally, the numberList represents the initial deck of cards. When later intervals are added this list would subsequently be altered.

The first function defined was the draw function. Generating a random number from index 0 to the length from the numberList, the function would return a randomized card. This simulated a shuffled deck. The index of the randomized card would then be removed.

The next function defined was called "checkEmpty". This function was crucial as it ensured that at all times one deck (or what was remaining after players drew) was being played, guaranteeing that the optimal strategy aforementioned was implemented. After each draw, the draw function was called; this function would be called to check whether or not the last draw was the last card that had not been played yet. If it was, the function would reset the deck by adding

```python
def draw():
    n = 0
    if len(numberList) > 0:
        index = random.randint(0, len(numberList)-1)
        n = numberList[index]
        numberList.pop(index)
    return n
```

Figure 3: The code segment above is the draw function.

back the original cards (a regular deck of cards whose values were defined in the introduction) and adding the current increment. For example, if the current increment was 0.25, 0.25 would be added to each deck.

```python
def checkEmpty(num):
    if len(numberList) == 0:
        for i in range (2, 10):
            for j in range (1, 5):
                numberList.append(i+(num/10))
        for k in range (1, 17):
            numberList.append(10+(num/10))
        for j in range (1, 5):
            numberList.append(11+(num/10))
```

Figure 4: The code segment above is the checkEmpty function. Note that the function takes in a "num", which simply signifies the current interval that is being used within the deck of cards. Thus, within each append function, "num" is added.

The strategy functions were the next to be created, essentially following the previously defined strategies for both the player and the house.

Finally, the win and reset functions were created to determine who (player or house) won each round, and to reset the lists that acted as the player's and house's hands. The win function simply checked whether or not an entity had busted. If neither the player or house had, it checked who had a closer sum to 21. The reset function cleared the player and house cards lists.

```
def win(a, b):
  numA = sum(a[0: len(a)])
  numB = sum(b[0: len(b)])
  if ((numA) > 21):
    global bustSum
    global bustNum
    bustSum = bustSum + len(playerCards)/2
    bustNum = bustNum + 0.5
    return "House"
  elif ((numB) > 21):
    return "Player"
  elif (numA > numB):
    return "Player"
  elif (numB > numA):
    return "House"
  else:
    return "tie"
```

Figure 5: The code segment returns who won by first checking whether either entity has busted. If not, it compares who has the higher total value. If both entities have the same total value, it returns a tie.

With the basic functions defined, the run function was created to combine all of the functions to simulate one round of black-jack between a single player and the house. After each round, variables would be added to keep track of the player wins, house wins, ties, cards needed to bust if the player did bust, and added onto the number of busts if the player did bust.

```python
def checkEmpty(num):
    if len(numberList) == 0:
        for i in range (2, 10):
            for j in range (1, 5):
                numberList.append(i+(num/10))
        for k in range (1, 17):
            numberList.append(10+(num/10))
        for j in range (1, 5):
            numberList.append(11+(num/10))
```

Figure 6: The code segment above is the function that runs one round of a game.

Finally, now that we were able to run one round of a game, we could loop through runGame thousands of times to find the player's winning chance and the average number cards to bust (calculated by dividing bustSum/bustNum). Since this study aimed to determine the correlation between changing the deck by a certain interval and the winning and average number of cards it took to bust, the for loop that wrapped the runGame function was then wrapped by another function that added an interval from 0 to 20 in increments of 0.01. Two graphs were then generated with winning rate and bust rates being the two response variables and the added increment being the explanatory variable.

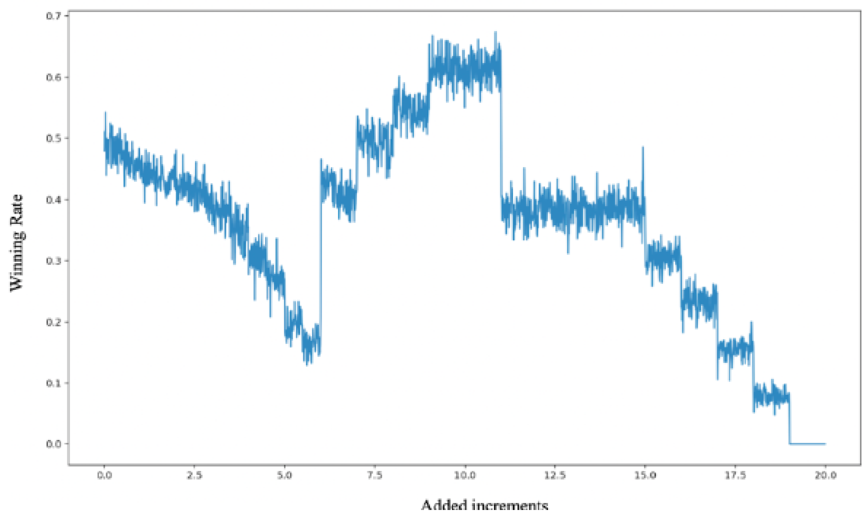

Figure 7: The graph shows the correlation between the increment value of the cards and the player's winning rate.

5 Data

To analyze the win rate data visually, we utilized the Matplotlib library to plot the lists. Figure 1 displays the correlation between the increment value of cards and the player's winning rate, with the increment value ranging from 0 to 20. The graph exhibits four noteworthy behaviors. Firstly, in several segments, the graph behaves similarly to a step function, where the intervals are densely packed together with similar winning rates. For example, (provide an interval as an example).

When adding increments from 0 to 6, the graph displays a decreasing linear graph. To prove this pattern, we implemented the concept of regression.When adding increments from 0 to 6, the graph displays a decreasing linear graph. To prove this pattern, we implemented the concept of regression. We analyzed both power and exponential by logging the x and y values, but there was a weak correlation. The r^2 value was significantly lower showing that there was a weak correlation between the x and y variables. The r^2 value represents the percent of the variation in the winning rate which can be explainable by the approximate linear relationship with the increments in the card values. We then analyzed a linear regression model between winning rate and card increments and the residual plots showed a clearer scatter of points. There was a slight curve but

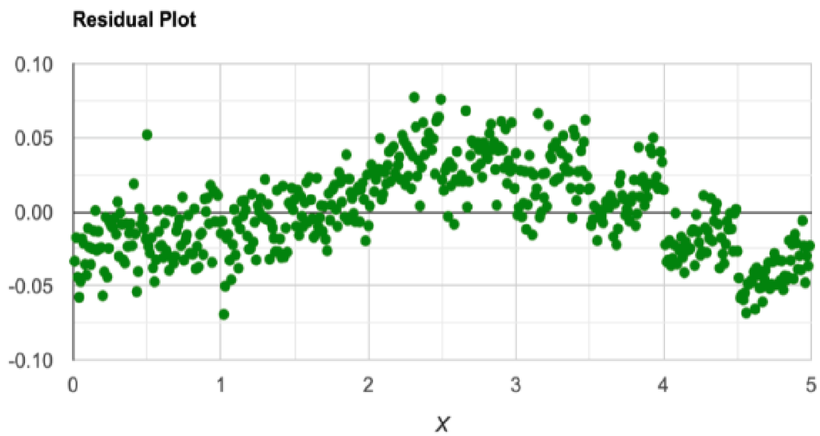

Figure 8: First Segment (0-6)

this model showed a more random pattern. The r^2 value was 0.89 representing the percent of the variation in the winning rate which can be explainable by the approximate linear relationship with the increments in the card values. The r value is 0.94 which shows a strong negative linear relationship between the winning rate and card increments. Therefore, this proved that this segment of the graph was a decreasing linear graph. As the card values increase in the standard deck, the player will have a higher chance of winning. As the card values are higher, the player will tend to bust more once hitting, thus decreasing their winning rate.

Second Segment (6-11) The graph exhibits a distinct pattern when adding increments of 6-11. During this segment, the graph shows a sharp increasing linear pattern. This is logical as adding these values will put the players in standing range. The optimal strategy is for players to stand when the sum of their cards is from 16-21, and adding these increments will tend to put the player in this range with the 2 cards they are dealt. Since the player is already in the standing range, they are less likely to bust and have a better hand than the house.

Third Segment (11-15) When adding increments from 11-15 to the standard deck, the graph displays a somewhat horizontal line at x = 0.4. This is because adding these increments puts the players at a sum where the player is supposed to stand. With a similar sum, the player will follow the same optimal strategy of what to do, leading the winning rate to be the same.

Fourth Segment (15-20) When adding increments from 15-20 to the standard deck, the graph displays a decreasing linear graph. This is logical since when the cards approach larger values, it will only take one more card for the sum of the players to exceed 21 (which will be shown in figure 2). Therefore, the winning rate will be much lower.

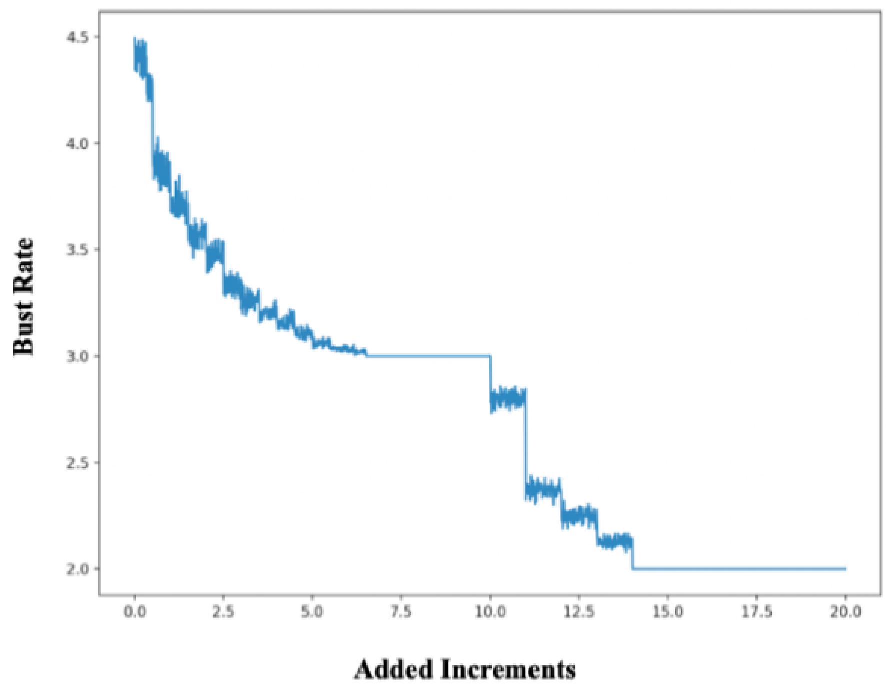

Figure 9: A similar method was utilized to generate the bust rate graph.Figure 2 illustrates the correlation between the increment value of the cards and the average number of cards it takes to bust. The increment value of cards ranges from 0 to 20.

Notably, it seems to be a decreasing power graph up to the increment of 10. It has an asymptote at y=3. This is logical since when the cards approach immensely large values, it only takes one more card for the sum of the players to exceed 21. This means that it will only take 2, the cards in the player's initial hand, plus one extra card for a total of 3 cards. Here is a second asymptote at x = 2. When the increments increase to such a large amount, the player's hand is considered an automatic bust. The graph also acts similarly to step function when the increments are positive. This means that within a set of intervals that are densely packed together, they have similar bust rates.

6 Limitations and Future Studies

In typical games, blackjack consists of multiple players and multiple decks. Our simulated rounds of blackjack used a single deck of cards and excluded rules such as betting, splitting pairs, doubling-down, and counting cards. To expand our analysis of blackjack player strategy with different numbers of decks, we could include splits, which allow players to split two identical cards into two separate hands. This would increase the player's chances of winning. We could also add monetary rewards after each round and determine the best strategy to maximize earnings, benefiting those who play blackjack at casinos. Furthermore, we could include more decks, as blackjack is usually played with multiple decks, which could lead to a more effective blackjack strategy. Additionally, coded card counting could be incorporated, where the player keeps track of high and low-valued cards dealt to determine when to bet more or less and change playing decisions based on the deck composition.

Moreover, we can analyze the graphs using other methods. Currently, we used power, linear, and exponential regression to check the correlation between bust and win rate. We could also further analyze why the graphs give the current shape and form, as of right now we are analyzing the graphs based on our current knowledge.

References

[1] Betway. (n.d.). Blackjack strategy 101: Double deck v single deck. Retrieved from https://blog.betway.com/casino/blackjack-strategy-101-double-deck-v-single-deck/

[2] Crescent University. (n.d.). The history of blackjack. Retrieved from https://crescent.edu/post/the-history-of-blackjack

[3] Elling, S. (2017, July 26). Number of decks and rules matter in blackjack. Review Fix. Retrieved from https://reviewfix.com/2017/07/number-decks-rules -matter-blackjack/: :text=

[4] Shackleford, M. (n.d.). Why number of decks matter in blackjack. Wizard of Odds. Retrieved from https://wizardofodds.com/games/blackjack/why-number -of-decks-matter/

[5] The World of Playing Cards. (n.d.). Blackjack. Retrieved from https://www.wopc.co.uk/history-the-history-of-playing-cards

[6] United States Playing Card Company. (n.d.). How to play blackjack. Retrieved from https://bicyclecards.com/how-to-play/blackjack/